$\underline{a} = (\cos t, \sin t)$

$\underline{b} = (\sin^2 t, \cos^2 t)$

a) $t = \frac{5\pi}{6}$

b) $t = \frac{5\pi}{6}$, $\alpha = ?$ (fokban, egészre)

α a és b szöge

c) $a \perp b \rightarrow t = ?$

a) $\underline{a} = \left(\cos\underbrace{\frac{5\pi}{6}}_{150^\circ}, \sin\frac{5\pi}{6}\right) = \left(\underbrace{-\frac{\sqrt{3}}{2}}_{a_1}, \underbrace{\frac{1}{2}}_{a_2}\right)$

-30°

$\underline{b} = \left(\sin^2\frac{5\pi}{6}, \cos^2\frac{5\pi}{6}\right) = \left(\left(\frac{1}{2}\right)^2, \left(-\frac{\sqrt{3}}{2}\right)^2\right) = \left(\underbrace{\frac{1}{4}}_{b_1}, \underbrace{\frac{3}{4}}_{b_2}\right)$

b) $|a| \cdot |b| \cdot \cos\alpha = a_1 \cdot b_1 + a_2 \cdot b_2 \rightsquigarrow 1 \cdot \frac{\sqrt{10}}{4} \cdot \cos\alpha = -\frac{\sqrt{3}}{8} + \frac{3}{8}$

$|a| = \sqrt{a_1^2 + a_2^2} = \sqrt{\left(-\frac{\sqrt{3}}{2}\right)^2 + \left(\frac{1}{2}\right)^2} = \sqrt{\frac{1}{4} + \frac{3}{4}} = \sqrt{1} = 1$

$|b| = \sqrt{b_1^2 + b_2^2} = \sqrt{\left(\frac{1}{4}\right)^2 + \left(\frac{3}{4}\right)^2} = \sqrt{\frac{10}{16}} = \frac{\sqrt{10}}{4}$

$\cos\alpha = \frac{3-\sqrt{3}}{2\sqrt{10}} \approx 0{,}2005$

$\alpha \approx 78{,}43^\circ \rightarrow \alpha = 78^\circ$ $\boxed{V}$

c) $\underline{a} \perp \underline{b}$ $\quad \underline{a}=(\cos t, \sin t)$ $\quad \underline{b}=(\sin^2 t, \cos^2 t)$ $\quad \langle a,b\rangle = a_1 \cdot b_1 + a_2 \cdot b_2$

$\Downarrow$

$\langle a,b\rangle = 0$

$$\langle a,b\rangle = \cos t \cdot \sin^2 t + \sin t \cdot \cos^2 t \overset{?}{=} 0$$

$$\sin t \cdot \cos t \cdot (\sin t + \cos t) = 0$$

$\sin t = 0$

$t = k\pi \quad k \in \mathbb{Z}$

$\cos t = 0$

$t = \frac{\pi}{2} + n\cdot\pi \quad n \in \mathbb{Z}$

$\sin t + \cos t = 0$

$\operatorname{tg} t = -1$

$t = \frac{3\pi}{4} + m\cdot\pi \quad m \in \mathbb{Z}$

$\Longrightarrow$ Tehát akkor lesz $\underline{a}$ és $\underline{b}$ $\perp$: $t_1 = n\cdot\frac{\pi}{2}$, $t_2 = \frac{3\pi}{4} + m\cdot\pi$ $\quad n,m \in \mathbb{Z}$.

$P(2;5)$ $\quad$ x y $\qquad x+y=4 \qquad x+y=6$ $\qquad$ metszőpontok első koordinátájainak különbsége 3

$y=4-x \qquad y=6-x$

$$y=mx+b$$

$$5=m\cdot 2+b \rightarrow b=5-2m \Rightarrow y=mx+5-2m$$

I. $\left.\begin{array}{l} y=mx+5-2m \\ y=4-x \end{array}\right\}$ $\qquad (m+1)x=2m-1 \qquad x_1=\frac{2m-1}{m+1} \qquad m\neq -1$

II. $\left.\begin{array}{l} y=mx+5-2m \\ y=6-x \end{array}\right\}$ $\qquad 6-x=mx+5-2m$

$(m+1)\cdot x=+1+2m \qquad x_2=\frac{2m+1}{m+1} \qquad m\neq -1$

1) $\frac{2m-1}{m+1}-\frac{2m+1}{m+1}=3 \quad = \frac{\cancel{2m}-1-\cancel{2m}-1}{m+1}=\frac{-2}{m+1} \qquad 3m+3=-2 \qquad m_1=-\frac{5}{3}$

2) $\frac{2m+1}{m+1}-\frac{2m-1}{m+1}=3 \quad = \frac{2m+1-2m+1}{m+1}=\frac{2}{m+1} \qquad 3m+3=2 \qquad m_2=-\frac{1}{3}$

$m_1 = -\frac{5}{3}$ $m_2 = -\frac{1}{3}$ $y = mx + b$ $P(2;5)$ $5 = 2m + b$

m_1: $5 = 2 \cdot \left(-\frac{5}{3}\right) + b_1$

$5 = -\frac{10}{3} + b_1 \rightsquigarrow b_1 = \frac{25}{3}$

m_2: $5 = 2 \cdot \left(-\frac{1}{3}\right) + b_2$

$5 = -\frac{2}{3} + b_2 \rightsquigarrow b_2 = \frac{17}{3}$

$\Rightarrow$ V: 2 egyenes elégíti ki a feltételeket:

$y = -\frac{5}{3}x + \frac{25}{3}$ $(\rightsquigarrow 5x + 3y = 25)$

$y = -\frac{1}{3}x + \frac{17}{3}$ $(\rightsquigarrow x + 3y = 17)$

I $\log_x(x^2y^3)+\log_y(x^3y)=9$

II $\cos(x+y)+\cos(x-y)=0$

$\boxed{\begin{array}{l} x,y\neq 1 \\ x,y>0 \end{array}}$

I $\log_x(x^2y^3)+\log_y(x^3y)=2+3\log_x y+1+3\log_y x=3+3(\log_x y+\log_y x)$

$\Rightarrow \underline{\log_x y}+\underline{\log_y x}=2$

reciprote $\Rightarrow 1 \Rightarrow x=y$

II $\cos 2x+\cos 0=0$

$\cos 2x=-1$

$2x=\pi+\underbrace{k2\pi} \Rightarrow x=\frac{\pi}{2}+k\pi \qquad k\in\mathbb{Z}$

V: $x=y=\frac{\pi}{2}+k\pi \qquad k\in\mathbb{N}$

$$\begin{cases} \text{I} \quad \log_x(x^2y^3)+\log_y(x^3y)=9 \\ \text{II} \quad \cos(x+y)+\cos(x-y)=0 \end{cases}$$

$$\boxed{\begin{matrix} x,y \neq 1 \\ x,y>0 \end{matrix}}$$

I $\quad \log_x(x^2y^3)+\log_y(x^3y)=2+3\log_x y+1+3\log_y x=3+3(\log_x y+\log_y x)$

$$\Rightarrow \underbrace{\log_x y+\log_y x}_{\text{reciprote} \Rightarrow 1}=2 \quad \Rightarrow x=y$$

II $\quad \cos 2x+\cos 0=0$

$\cos 2x=-1$

$2x=\pi+k2\pi \Rightarrow x=\frac{\pi}{2}+k\pi \qquad k\in\mathbb{Z}$

V: $\quad x=y=\frac{\pi}{2}+k\pi \qquad k\in\mathbb{N}$

a) $\dfrac{2x^2+x-10}{2^{x-1}-2}=0$ b) $\sqrt{x+16}+\sqrt{x-9}=5$ $\rightsquigarrow$ 1 mo.

c) $\lg(x^2+x-6)=\lg(1-x^2)$ d) $\sin x-1=\sqrt{\lg(\cos^2 x-1,5\cos x)}$ $\rightsquigarrow$ $\emptyset$ mo.

a) $2^{x-1}-2\neq 0$

$2^{x-1}\neq 2$ $x\neq 2$

$2x^2+x-10=0$ $x_{1,2}=\dfrac{-b\pm\sqrt{b^2-4ac}}{2a}=\dfrac{-1\pm\sqrt{1-4\cdot 2\cdot(-10)}}{2\cdot 2}=$

$=\dfrac{-1\pm\sqrt{81}}{4}=\dfrac{-1\pm 9}{4}=$ $x_1=2$ → nem mo. mert $x\neq 2$

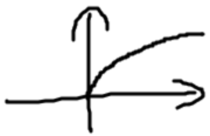
$\boxed{x_2=-\frac{5}{2}}$

b) $x+16\geq 0$ $x-9\geq 0$

$x\geq -16$ $x\geq 9$

1. mo.: $f(x)=\sqrt{x+16}+\sqrt{x-9}$ $x=9$

f szig. mon. nő → $\exists$ min. $f(9)=5$



2. mo.

$x+16+x-9+2\cdot\sqrt{(x+16)\cdot(x-9)}=25$

$10\sqrt{x-9}=0$ $\Rightarrow x=9$

Ell:

V:

a) $\dfrac{2x^2+x-10}{2^{x-1}-2}=0$

b) $\sqrt{x+16}+\sqrt{x-9}=5 \rightsquigarrow 1$ mo.

c) $\lg(x^2+x-6)=\lg(1-x^2)$

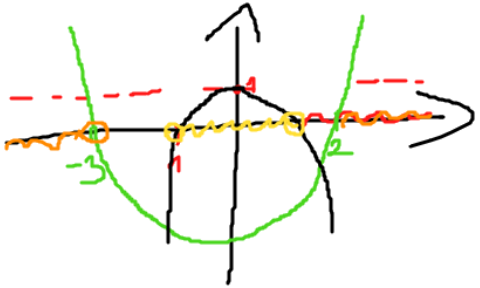
d) $\sin x-1=\sqrt{\lg(\cos^2 x-1{,}5\cos x)} \rightsquigarrow \emptyset$ mo.

c) $x^2+x-6>0$

$1-x^2>0$

$x^2+x-6=0$

$x_{1,2}=\dfrac{-1\pm\sqrt{1-4\cdot1\cdot(-6)}}{2}=\dfrac{-1\pm\sqrt{25}}{2}=\begin{cases}x_1=2\\ x_2=-3\end{cases}$



A: $-1<x<1$

B: $x<-3$ vagy $x>2$

$A\cap B=\emptyset \Rightarrow \emptyset$ mo.

d) $\sin x-1=\sqrt{\lg(\cos^2 x-1{,}5\cos x)}$

$\lg(\cos^2 x-1{,}5\cos x)\geq 0$

$\underbrace{\cos^2 x}_{0}-1{,}5\underbrace{\cos x}_{0}>0 \rightsquigarrow x\geq 0$

$\Rightarrow \sin x-1\geq 0 \quad \sin x\geq 1 \quad x=\dfrac{\pi}{2}+2k\pi \quad (k\in\mathbb{Z})$

$\cos\left(\dfrac{\pi}{2}+2k\pi\right)=0 \rightsquigarrow \emptyset$ mo.

$$\overbrace{1}^{a}x^2 - \overbrace{(4p+1)}^{b}x + \overbrace{2p}^{c} = 0 \qquad p \in \mathbb{R}$$

a) 2 valós gyök b) $x_1 = 3 \to x_2 = ?$ c) $x_1^2 + x_2^2 = 7$, $p = ?$

a) $D > 0 \quad D = b^2 - 4ac \quad (4p+1)^2 - 4 \cdot 1 \cdot 2p = 16p^2 + 8p + 1 - 8p = 16p^2 + 1 > 0 \rightsquigarrow$ 2 mo.

b) $x = 3$: $3^2 - (4p+1) \cdot 3 + 2p = 0 \quad 9 - 12p - 3 + 2p = 0 \quad 6 - 10p = 0 \quad p = 0{,}6$

$x^2 - 3{,}4x + 1{,}2 = 0 \rightsquigarrow$ 2 féle mo.: $x_{1,2} = \frac{3{,}4 \pm \sqrt{3{,}4^2 - 4 \cdot 1 \cdot 1{,}2}}{2 \cdot 1} = \begin{cases} x_1 = 3 \\ x_2 = 0{,}4 \end{cases}$

V:

c) $x_1^2 + x_2^2 = (x_1 + x_2)^2 - 2x_1x_2 \qquad x_1 + x_2 = 4p+1 \qquad x_1 \cdot x_2 = 2p$

$\Rightarrow x_1^2 + x_2^2 = (4p+1)^2 - 2 \cdot 2p = 7$

$16p^2 - 8p + 1 - 4p = 7$

$16p^2 - 12p + 1 = 7$

$16p^2 - 12p - 6 = 0$

$p_{1,2} = \frac{12 \pm \sqrt{(-12)^2 - 4 \cdot 16 \cdot (-6)}}{2 \cdot 16} = \begin{cases} p_1 = 0{,}5 \\ p_2 = -0{,}75 \end{cases}$

V:

a) $x, y > 0 \quad x, y \in \mathbb{R}$

$$\left.\begin{aligned} x+y &= 0{,}2 \\ \frac{\lg x + \lg y}{2} &= \lg \frac{x+y}{2} \end{aligned}\right\} \longrightarrow \boxed{x = 0{,}2 - y}$$

$$\frac{\lg(0{,}2-y) + \lg y}{2} = \underbrace{\lg 0{,}1}_{-1}$$

$$\lg\big((0{,}2-y)y\big) = \underbrace{-2}_{\lg 0{,}01} \overset{\text{def.}}{\rightsquigarrow} (0{,}2-y)y = 0{,}01 \qquad y^2 - 0{,}2y + 0{,}01 = 0$$

$$y_{1,2} = \frac{0{,}2 \pm \sqrt{(-0{,}2)^2 - 4\cdot 1\cdot 0{,}01}}{2\cdot 1} = 0{,}1 \rightsquigarrow x = 0{,}1$$

Ell!

V!

b) $[-\pi, \pi]$ $\quad 2\sin^2 x - \cos x = 2$

$\sin^2 x + \cos^2 x = 1$

$\rightsquigarrow \sin^2 x = 1 - \cos^2 x$

$$2(1-\cos^2 x) - \cos x = 2$$

$$2\cos^2 x + \cos x = 0$$

$$\cos x(2\cos x + 1) = 0$$

$\cos x = 0$

$\downarrow$

$x = -\frac{\pi}{2}$

$x = \frac{\pi}{2}$

$2\cos x + 1 = 0$

$\cos x = -\frac{1}{2}$

$\downarrow$

$x = \frac{-2\pi}{3}$

$x = \frac{2\pi}{3}$

Ell?

v?

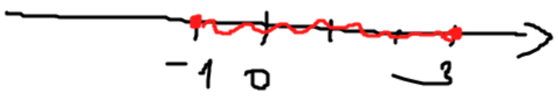
2021. május

ÉT: $[-1,3]$

1) a) $\sqrt{-2x+6} = x+1 \quad /(\ )^2$ $\qquad -2x+6 \geq 0 \qquad 2x \leq 6 \qquad x \leq 3$

$x+1 \geq 0 \qquad x \geq -1$

R:



$-2x+6 = x^2+2x+1$

$x^2+4x-5=0$

$$x_{1,2} = \frac{-4 \pm \sqrt{4^2-4\cdot 1\cdot(-5)}}{2\cdot 1} = \frac{-4\pm\sqrt{36}}{2} = \frac{-4\pm 6}{2} = \begin{cases} x_1 = 1 \in ÉT \\ x_2 = -5 \notin ÉT \end{cases}$$

$\rightsquigarrow$ nem mo.

V: $x=1$

b) $2\log_4 x^2 + 3\log_4 x^3 = \log_4 x^4 + \log_4 8^9 \qquad x>0$

$\cancel{4\log_4 x} + 9\log_4 x = \cancel{4\log_4 x} + 9\log_4 8$

$\log_4 x = \log_4 8 \xrightarrow[\text{(kölcs. egyért.)}]{\text{def}} x = 8$

Ell.?

V?